Scope of this project is to study quasilinear parabolic equations in step 2 sub Riemannian structures. The corresponding sub elliptic problem has been studied in [4] and [3]. The simplest space of this type is the Heisenberg group, whose Lie algebra has two generators and step 2.

In the elliptic case X. Zhong [4] established the C^{1, \alpha} regularity of the gradient in the Heisenberg group, by means of some delicate Cacciopoli type inequalities, and by De Giorgi method. A. Domokos, J. J. Manfredi established a preliminary result for step 3 groups. The difficulty is related to the fact that the vector fields are non-commutative, so that a mixed Cacciopoli inequality is needed. The same result, was extended to a more general setting of sub elliptic operators of step 2 by Citti and Mukherjee. In the subriemannian parabolic setting for the p-Laplacian there exists a preliminary result of Capogna Citti Zhong [1] under the assumption that 2<p<4, in the Heisenberg group, where the boundness of the gradient is studied.

We would be interested to extend the parabolic result to the general setting of 2 step Carnot group. We will probably need to use non linear balls, depending on the solution itself, which can balance the fact that the operator has a different homogeneity in the t variable with respect to the x variable. The problem is more delicate than in the known case since in general the vector fields will not be nilpotent, and new terms can show up

The researcher is expected to travel within the GHAIA project for a significative number of months and visit colleagues expert on the topic of the project to obtain the result or to study its application

[1] Luca Capogna, Giovanna Citti, Xiao Zhong, Lipschitz regularity for solutions of the parabolic p-Laplacian in the Heisenberg group, https://arxiv.org/abs/2106.05998

[2] A. Domokos, J. J. Manfredi C1;_-subelliptic regularity on SU(3) and compact, semi-simple Lie groups, Analysis and Mathematical Physics 10 (1) (2020).

[3] S. Mukherjee, X. Zhong, C1;_ regularity for variational problems in the Heisenberg group. Analysis and PDE, 14 (2), 2021, 567-594.

[4] X. Zhong, Regularity for variational problems in the Heisenberg group, preprint 2009. arXiv:1711.03284